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الأربعاء

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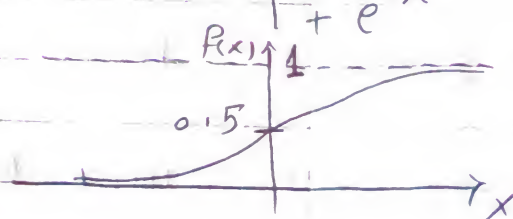
محاضرة [3]

Sigmoidal functions in neural networks

Sigmoidal functions

Binary Sigmoid

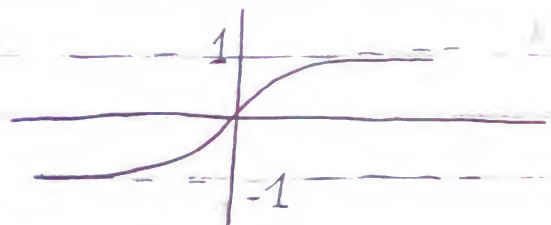
$$f(x) = \frac{1}{1 + e^{-x}}$$



Bipolar Sigmoid

$$g(x) = \frac{2}{1 + e^{-x}} - 1$$

$$g(x) = \frac{e^{-x} - 1}{e^{-x} + 1}$$



issues with Binary threshold



- ① abrupt change
- ② not differentiable
- ③ at zero, we cannot determine $f(x)$, some assume it to be 0.5, zero, 1
- ④ not continuous
- ⑤ function is not reversible, if you know $S=1$, you cannot know the value of y since $0 < y < \infty$

Binary Sigmoidal function: $f(x) = \frac{1}{1 + e^{-x}}$

problem 1:

sketch the graph of the binary sigmoidal function

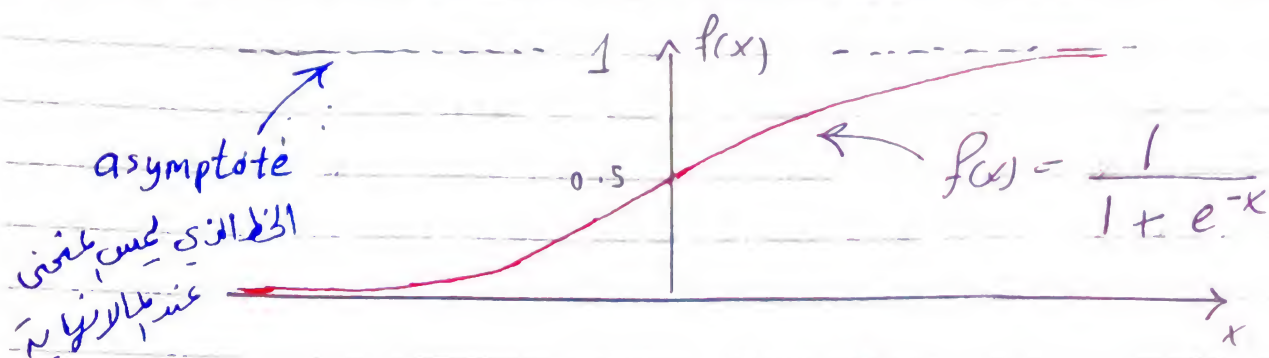
$$f(x) = \frac{1}{1 + e^{-x}}$$

verify that $0 < f(x) < 1$

$$f(-x) = 1 - f(x)$$

calculate $f(0)$; $f(2)$; $f(-2)$

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
f(x)	0.007	0.018				0.5				0.982	0.993



~~we~~ we have 2 asymptotes: ① $f(x) = 1$; $f(x) = -1$

to verify, we can sketch or substitute numerically

by me { $f(0) = \frac{1}{1+e^0} = \frac{1}{2}$, $f(\infty) = \frac{1}{1-0} = 1$

$f(-\infty) = \frac{1}{\infty} = 0 \Rightarrow 0 < f(x) < 1$

$f(-4) = 0.018 = 1 - f(x)^4 = 1 - 0.982 = 0.018$

OR

$$1 - f(x) = 1 - \frac{1}{1+e^{-x}} = \frac{1+e^{-x}-1}{1+e^{-x}} = \frac{e^{-x}}{1+e^{-x}} = \frac{1}{e^x+1}$$

$$= \frac{1}{1+e^{(-x)}} = f(-x)$$

* calculate ;

① $f(0) = \frac{1}{1+e^0} = \frac{1}{2}$

② $f(2) = \frac{1}{1+e^2} = 0.881$

③ $f(-2) = 1 - 0.881 = 0.119$
 $= \frac{1}{1+e^2} = 0.119$

Problem 2:-

Convince yourself that $f(x)$ is bounded, continuous, monotonically increasing, differentiable at all points.

- (1) bounded by 1 and 0
- (2) continuous \rightarrow smooth continuous curve
- (3) monotonically increasing, $f(x)$ increases with increasing x
- (4) we ~~can~~ we can draw tangent line at any point

Problem 3:-

Verify that $x = \ln f(x) - \ln [1 - f(x)]$
 $= \ln \left[\frac{f(x)}{1 - f(x)} \right]$

* يعني آخر البيت انك بتدري معرفة $f(x)$ معرفة x reversible اي انك بتدري معرفة x بتدري معرفة $f(x)$

$f(x) = \frac{1}{1 + e^{-x}}$ $f(x) + e^{-x} f(x) = 1$ $e^{-x} f(x) = 1 - f(x)$ $e^{-x} = \frac{1 - f(x)}{f(x)}$	$-x = \ln \left[\frac{1 - f(x)}{f(x)} \right]$ $x = \ln \left[\frac{f(x)}{1 - f(x)} \right]$ $= \ln f(x) - \ln [1 - f(x)]$
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* calculate x if $f(x) = 0.75$

$$x = \ln \left[\frac{0.75}{1 - 0.75} \right] = 1.099$$

Problem 4:
verify that

$$\frac{df(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} \quad \text{①}$$

لست استخدم هذا القانون عندما
تد تكون معروفة

OR alternatively ; $\frac{df(x)}{dx} = f(x)[1-f(x)]$ ②

سأستخدم هذا القانون عندما $f(x)$ معروفة

$$f(x) = \frac{1}{1+e^{-x}}$$

$$\frac{df(x)}{dx} = -\frac{e^{-x}}{(1+e^{-x})^2} \quad \checkmark \quad \text{①}$$

ويمكن التعبير عن هذا بالتفاضل بالطريقة التالية :

$$\begin{aligned} \frac{df(x)}{dx} &= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}} = \frac{1}{1+e^{-x}} \cdot \frac{1+e^{-x}-1}{1+e^{-x}} \\ &= \frac{1}{1+e^{-x}} \left[1 - \frac{1}{1+e^{-x}} \right] \\ &= f(x) [1 - f(x)] \end{aligned}$$

Problem 5:-

السطح البياني للتفاضل لـ x

① sketch $\frac{df(x)}{dx}$; prove that the graph is

Symmetric about $x=0$ axis

② prove that $\frac{df(x)}{dx}$ has a maximum value of

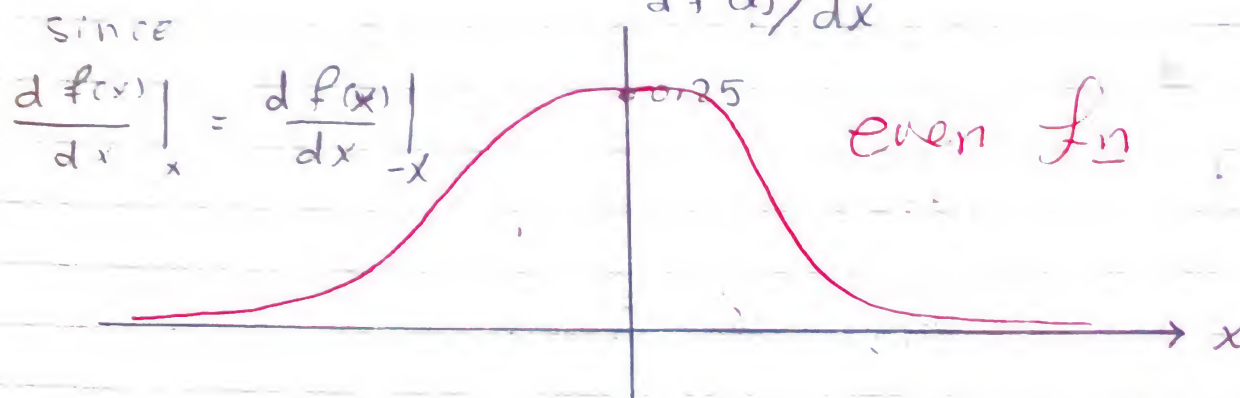
0.25 at $x=0$;

③ calculate $\left. \frac{df(x)}{dx} \right|_{x=2}$; $\left. \frac{df(x)}{dx} \right|_{x=-2}$

$$\frac{df(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2}$$

1	-5	-4	-3	-2	-1	0	1	2	3	4	5
$\frac{df(x)}{dx}$	0.007	0.018				0.25				0.018	0.007

$\frac{df(x)}{dx}$ is even function \rightarrow symmetric



① الدالة زوجية (even) حيث أنها متماثلة حول المحور الرأسي

② لهذا الرقم قيمة عظمى تساوي 0.25 وقدت عندما $x=0$

$$\left. \frac{df(x)}{dx} \right|_{x=2} = 0.105 ; \left. \frac{df(x)}{dx} \right|_{x=-2} = 0.105$$

Problem 6 -

for a given value of $\frac{df(x)}{dx}$ verify that there are two values of x ; x_1 and x_2 such that

$$x_1 + x_2 = 0$$

$$f(x_1) + f(x_2) = 1$$

calculate $x_1, x_2, f(x_1), f(x_2)$ when $\frac{df(x)}{dx} = 0.15$

$$f(x_2) = 1 - f(x_1) \quad [\text{from problem 1}]$$

$$f(x_1) + f(x_2) = 1$$

يمكن إثبات النتائج السابقة بطريقة أخرى كما يلي:

from problem 4

$$\frac{d f(x)}{d(x)} = f(x) [1 - f(x)]$$

$$\frac{d f(x)}{d(x)} = f(x) - f^2(x)$$

$$f^2(x) - f(x) + \frac{d f(x)}{d x} = 0$$

معادلة من الدرجة الثانية في x ، فيصبح هناك جذران للحدادة
two roots

$$f_1(x) = f(x_2) \quad ; \quad f_2(x) = f(x_2)$$

$$f_1(x) + f_2(x) = -\frac{b}{a} = -\frac{(-1)}{1} = 1$$

$$\therefore f(x_1) + f(x_2) = 1$$

$$\ast \quad \frac{1}{1 + e^{-x_1}} + \frac{1}{1 + e^{-x_2}} = 1$$

$$1 + e^{-x_2} + 1 + e^{-x_1} = (1 + e^{-x_1})(1 + e^{-x_2})$$

$$2 + \cancel{e^{-x_1}} + \cancel{e^{-x_2}} = 1 + \cancel{e^{-x_1}} + \cancel{e^{-x_2}} + e^{-(x_1+x_2)}$$

$$e^{-(x_2+x_1)} = 1 \quad \ln$$

$$-x_2 - x_1 = \ln 1 = 0$$

$$x_1 + x_2 = 0$$

* given $\frac{df(x)}{dx} = 0.15$

$$f^2(x) - f(x) + 0.15 = 0$$

$$f(x_1) = \frac{1 + \sqrt{1 - 4(0.15)}}{2} = 0.816$$

$$f(x_2) = \frac{1 - \sqrt{1 - 4(0.15)}}{2} = 0.184$$

$$= 1 - f(x_1)$$

$$x_1 = \ln \left[\frac{f(x_1)}{1 - f(x_1)} \right] \quad \text{given } f(x_1)$$

$$= \ln \left[\frac{0.816}{1 - 0.816} \right] = 1.489$$

$$x_2 = -x_1 = \ln \left[\frac{f(x_2)}{1 - f(x_2)} \right] = \ln \left[\frac{0.184}{1 - 0.184} \right]$$

$$= -1.489$$

Problem 7:-

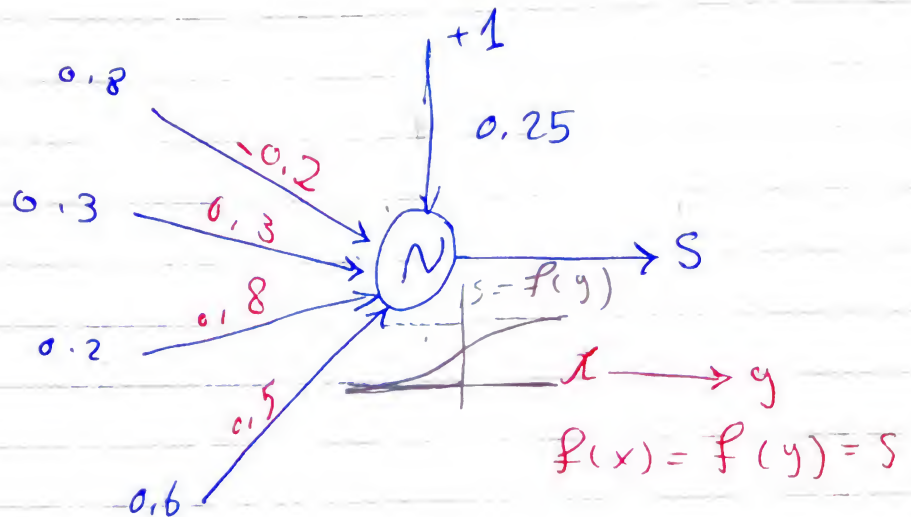
4 input Binary sigmoidal neuron f_n عنه neuron يستعمل Binary sigmoidal

$$x_1 = 0.8; x_2 = 0.3; x_3 = 0.2; x_4 = 0.6$$

$$w_1 = 0.2; w_2 = 0.3; w_3 = 0.8; w_4 = 0.5$$

بالترتيب على الترتيب

اصب الخرج اذا كان Bias weight ب 0.25



$$S = f(y) = \frac{1}{1 + e^{-y}}$$

Activation (y)

$$y = (0.8)(-0.2) + (0.3)(0.3) + (0.2)(0.8) + (0.6)(0.5) + (1)(0.25)$$

Bias

$$= 0.64$$

neuron output for a binary sigmoidal f_n

$$S = \frac{1}{1 + e^{-y}} = \frac{1}{1 + e^{-0.64}} = \boxed{0.655}$$

Problem 8

عندنا neuron بـ "BSF" Binary Sigmoidal function: y activation عندنا

يكون output S بـ 0.44

نحسب العزم وخطوب y (عكس بـ سابقه)

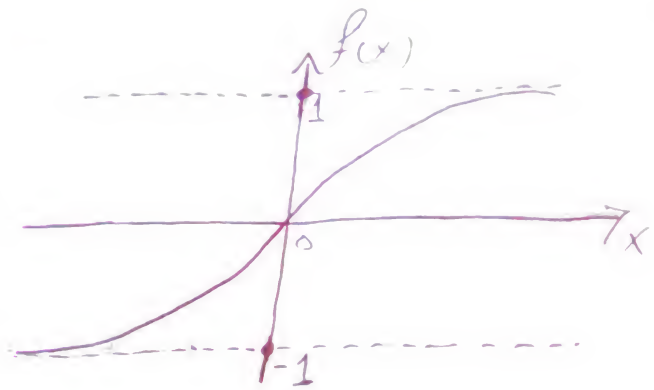
$$y = \ln \left[\frac{S}{1-S} \right] \quad [0 < S < 1] \quad \text{نرط}$$

$$y = \ln \left[\frac{0.44}{1-0.44} \right] = \boxed{-0.241}$$

Bipolar Sigmoidal f_n

$$g(x) = \frac{2}{1+e^{-x}} - 1$$

$$= \frac{1-e^{-x}}{1+e^{-x}}$$



$$g(x) = \frac{2}{1+e^{-x}} - 1$$

$$= \frac{2 - 1 - e^{-x}}{1 + e^{-x}} = \frac{1 - e^{-x}}{1 + e^{-x}} \quad [\text{Proof}]$$

Problem 9

Sketch the bipolar sigmoidal f_n .

$$g(x) = \frac{2}{1+e^{-x}} - 1$$

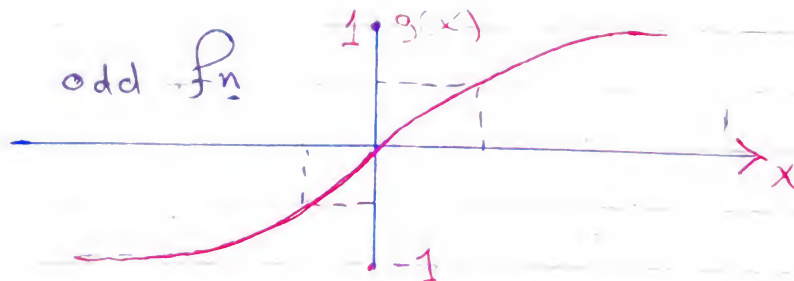
Verify that

$$-1 < g(x) < 1$$

$$g(-x) = -g(x)$$

Calculate $g(0)$, $g(2)$, and $g(-2)$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$g(x)$	-0.984	-0.944				0				0.964	0.986



$$g(-x) = -g(x)$$

هذه اثبات العكس
بالطريقة الثانية

$$g(x) = \frac{2}{1+e^{-x}} - 1 \rightarrow g(-x) = \frac{2}{1+e^x} - 1$$

$$\begin{aligned}
 g(-x) &= \frac{2}{1+e^x} - 1 = \frac{2-1-e^x}{1+e^x} \\
 &= \frac{1-e^x}{1+e^x} = \frac{e^{-x}-1}{e^{-x}+1} = \frac{e^{-x}-2+1}{e^{-x}+1} \\
 &= \frac{-2}{1+e^{-x}} + 1 = -\left[\frac{2}{1+e^{-x}} - 1\right] \\
 &= -g(x)
 \end{aligned}$$

$$g(0) = \frac{2}{1+e^0} - 1 = 1 - 1 = 0$$

$$g(2) = \frac{2}{1+e^{-2}} - 1 = 0.762$$

$$g(-2) = -g(2) = -0.762 = \frac{2}{1+e^2} - 1$$

Problem 10

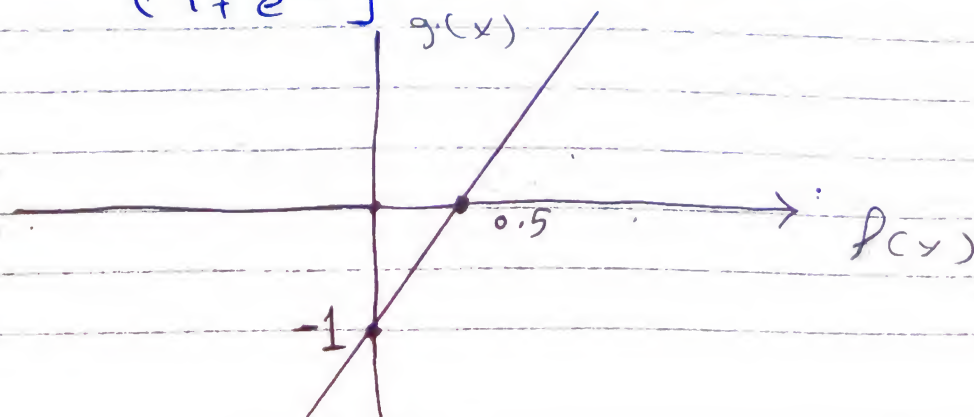
* Verify that $g(x) = 2f(x) - 1$

* Represent the relation graphically,

* What is the value of $g(x)$ when $f(x) = 0.5$?

* What is the value of $f(x)$ when $g(x) = -1$?

$$\begin{aligned}
 f(x) &= \frac{1}{1+e^{-x}} ; g(x) = \frac{2}{1+e^x} - 1 \\
 &= 2 \left[\frac{1}{1+e^x} \right] - 1 = 2f(x) - 1
 \end{aligned}$$



$$f(x) = 0.5 \rightarrow g(x) = 2(0.5) - 1 = 0$$

$$g(x) = -1 \rightarrow f(x) = \frac{1-1}{2} = 0$$

$f(x)$ و $g(x)$ لهما نفس القيمة

Problem 11

verify that

$$* \frac{dg(x)}{dx} = \frac{2e^{-x}}{(1+e^{-x})^2} = 0.5[1-g^2(x)]$$

\swarrow x \swarrow $g(x)$

$$* \frac{dg(x)}{dx} = 2 \frac{df(x)}{dx} ; \text{ sketch الجرافة } f(x) \text{ و } g(x)$$

$$* \text{ calculate } \left. \frac{dg(x)}{dx} \right|_{x=2} \text{ and } \left. \frac{dg(x)}{dx} \right|_{x=-2}$$

$$g(x) = \frac{2}{1+e^{-x}} - 1 = \frac{1-e^{-x}}{1+e^{-x}}$$

$$\frac{dg(x)}{dx} = \frac{(1+e^{-x})(e^{-x}) - (1-e^{-x})(-e^{-x})}{(1+e^{-x})^2}$$

$$= \frac{e^{-x} + e^{-2x} + e^{-x} + e^{-2x}}{(1+e^{-x})^2} = \frac{2e^{-x}}{(1+e^{-x})^2}$$

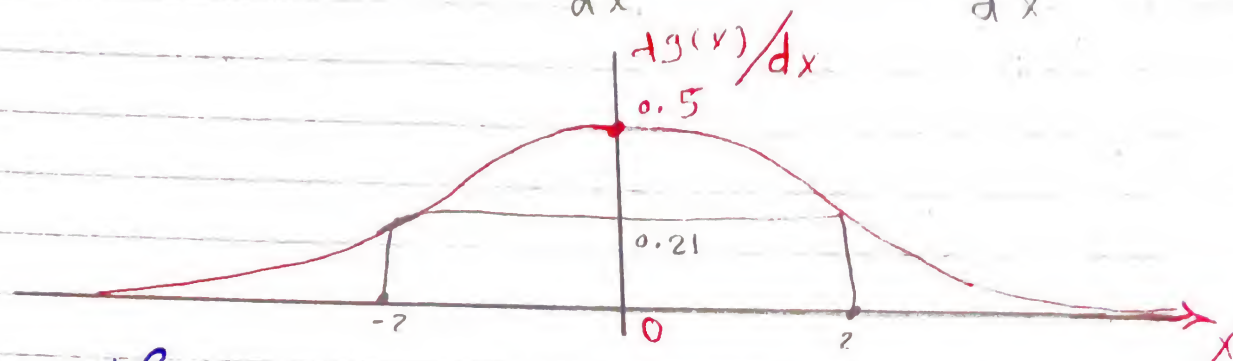
From problem 4 we conclude $\frac{dg(x)}{dx} = 2 \frac{df(x)}{dx}$

$$\frac{dg(x)}{dx} = \frac{2e^{-x}}{(1+e^{-x})^2} = 0.5 \left[\frac{4e^{-x}}{(1+e^{-x})^2} \right]$$

$$= 0.5 \left[\frac{(1+e^{-x})^2 - (1-e^{-x})^2}{(1+e^{-x})^2} \right] = 0.5[1-g^2(x)]$$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$dg(x)/dx$	0.014	0.036				0.5				0.036	0.014

⑤ لاظهار قيم $\frac{dg(x)}{dx}$ هي ضعف قيم $\frac{df(x)}{dx}$ المتأخرة في الجدول لكل حالة



* القيمة العظمى للتفاضل هي 0.5 عند $x=0$ حيث يتبعها ينحني $\frac{df(x)}{dx}$ في

حالة 5، ونجد $\frac{dg(x)}{dx}$ متأخر حول محور $x=0$

* قيمة $\frac{dg(x)}{dx}$ الى صفر عند تأويل x الى $-\infty, \infty$

$$\left. \frac{dg(x)}{dx} \right|_{x=2} = \left. \frac{dg(x)}{dx} \right|_{x=-2} = 0.21$$

Problem 12

A Neuron has a bipolar sigmoidal function,
Derive an expression for the activation y
in terms of the neural outputs

Calculate y for $s=0.6$

and for y for $s=-0.6$

$$s = \frac{2}{1 + e^{-y}} - 1 = \frac{1 - e^{-y}}{1 + e^{-y}}$$

$$s + s e^{-y} = 1 - e^{-y}$$

$$e^{-y} = \frac{1-s}{1+s}$$

$$-y = \ln \left[\frac{1-s}{1+s} \right]$$

$$y = \ln \left[\frac{1+s}{1-s} \right] = \ln(1+s) - \ln(1-s)$$

$$y = \ln \left[\frac{1+0.6}{1-0.6} \right] = 1.386 \quad -1 < s < 1$$

$$s = -0.6 \rightarrow s = \ln \left[\frac{1-0.6}{1+0.6} \right] = -1.386$$